

NEW CONCEPTS OF NEUTROSOPHIC SETS

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ABSTRACT

In this paper we will introduce and study some types of neutrosophic sets (NS for short). Finally, we extend the concept of intuitionistic fuzzy ideal [8] to the case of neutrosophic sets. We can use the new of neutrosophic notions in the following applications: compiler, networks robots, codes and database.

KEYWORDS: Fuzzy Set, Intuitionistic Fuzzy Set, Neutrosophic Set, Intuitionistic Fuzzy Ideal, Neutrosophic Ideal

1-INTRODUCTION

The neutrosophic set concept was introduced by Smarandache [11, 12]. In 2012 neutrosophic sets have been investigated by Hanafy and Salama at el [4, 5, 6, 7, 8, 9]. The fuzzy set was introduced by Zadeh [13] in 1965, where each element had a degree of membership. In 1983 the intuitionistic fuzzy set was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. Salama at el [8] defined intuitionistic fuzzy ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [10]. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts. In this paper we will introduce the definitions of normal neutrosophic set, convex set, the concept of α -cut and neutrosophic ideals (NL for short), which can be discussed as generalization of fuzzy and fuzzy intuitionistic studies.

2-TERMINOLOGIES

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [11, 12], and Salama et al. [4, 5, 6, 7, 8].

3-SOME TYPES OF NEUTROSOPHIC SETS

Definition.3.1

A neutrosophic set A with $\mu_A(x) = 1$, or $\sigma_A(x) = 1$, $\gamma(x) = 1$ is called normal neutrosophic set.

In other words A is called normal if and only if $\max_{x \in X} \mu_A(x) = \max_{x \in X} \sigma_A(x) = \max_{x \in X} \gamma_A(x) = 1$.

Definition.3.2

When the support set is a real number set and the following applies for all $x \in [a, b]$ over any interval $[a, b]$:

$$\mu_A(x) \geq \mu_A(a) \wedge \mu_A(b) \quad ; \quad \sigma_A(x) \geq \sigma_A(a) \wedge \sigma_A(b) \quad \text{and} \quad \gamma_A(x) \geq \gamma_A(a) \wedge \gamma_A(b)$$

A is said to be convex.

Definition 3.3

When $A \subset X$ and $B \subset Y$, the neutrosophic subset $A \times B$ of $X \times Y$ that can be arrived at the following way is the direct product of A and B.

$$A \times B \leftrightarrow \mu_{A \times B}(x, y) = \mu_A(x) \wedge \mu_B(x)$$

$$\sigma_{A \times B}(x, y) = \sigma_A(x) \wedge \sigma_B(x)$$

$$\gamma_{A \times B}(x, y) = \gamma_A(x) \wedge \gamma_B(x)$$

We must first introduce the concept of α -cut

Definition 3.4

For a neutrosophic set $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$

$$A_\alpha = \left\{ x : x \in X, \text{either } \mu_A(x), \sigma_A(x) > \alpha \text{ or } \nu_A(x) < 1 - \alpha \right\}; \alpha \in \left] 0, 1 \right[$$

$$A_{\bar{\alpha}} = \left\{ x : x \in X, \text{either } \mu_A(x), \sigma_A(x) \geq \alpha \text{ or } \nu_A(x) \leq 1 - \alpha \right\}; \alpha \in \left] 0, 1 \right[$$

are called the weak and strong α -cut respectively.

Making use α -cut, the following relational equation is called the resolution principle.

Theorem 3.1

$$\mu_A(x) = \sigma_A(x) = \gamma_A(x) = \text{Sup}_{x \in \left] 0, 1 \right[} \left[\alpha \wedge \chi_{A_\alpha}(x) \right]$$

$$\mu_A(x) = \sigma_A(x) = \gamma_A(x) = \text{Sup} \left[\alpha \wedge \chi_{A_{\bar{\alpha}}}(x) \right]$$

Proof

$$\text{Sup}_{x \in \left] 0, 1 \right[} \left[\alpha \wedge \chi_{A_{\bar{\alpha}}}(x) \right] = \text{Sup}_{\alpha \in \left\{ \begin{array}{l} \left(\begin{array}{l} - \\ 0, \mu_{A_{\bar{\alpha}}}(x) \end{array} \right) \\ \left(\begin{array}{l} - \\ 0, \sigma_{A_{\bar{\alpha}}}(x) \end{array} \right) \\ \left(\begin{array}{l} - \\ 0, \gamma_{A_{\bar{\alpha}}}(x) \end{array} \right) \end{array} \right\}} \left[\alpha \wedge \chi_{A_{\bar{\alpha}}}(x) \right] = \text{Sup}_{\alpha \in \left\{ \begin{array}{l} \left(\begin{array}{l} + \\ \mu_{A_{\bar{\alpha}}}(x), 1 \end{array} \right) \\ \left(\begin{array}{l} + \\ \sigma_{A_{\bar{\alpha}}}(x), 1 \end{array} \right) \\ \left(\begin{array}{l} + \\ \gamma_{A_{\bar{\alpha}}}(x), 1 \end{array} \right) \end{array} \right\}} \left[\alpha \wedge \chi_{A_{\bar{\alpha}}}(x) \right]$$

$$= \text{Sup}_{\alpha \in (0, \mu_A(x))} \left[\alpha \wedge 1 \right] \vee \text{Sup}_{\alpha \in (0, \sigma_A(x))} \left[\alpha \wedge 1 \right] \vee \text{Sup}_{\alpha \in (0, \gamma_A(x))} \left[\alpha \wedge 1 \right]$$

$$= \text{Sup}_{\alpha \in \left\{ \begin{array}{l} \left(\begin{array}{l} - \\ 0, \mu_A(x) \end{array} \right) \\ \left(\begin{array}{l} - \\ 0, \sigma_A(x) \end{array} \right) \\ \left(\begin{array}{l} - \\ 0, \gamma_A(x) \end{array} \right) \end{array} \right\}} \alpha = \mu_A(x) = \sigma_A(x) = \gamma_A(x)$$

If we defined the neutrosophic set αA_α here as

$$\alpha A_\alpha \leftrightarrow \mu_{\alpha A_\alpha} = \alpha \wedge \chi_{A_{\bar{\alpha}}}(x) = \sigma_{\alpha A_{\bar{\alpha}}}(x) = \gamma_{\alpha A_{\bar{\alpha}}}(x)$$

The resolution principle is expressed in the form

$$A = \bigcup_{\alpha \in \left[\begin{smallmatrix} - \\ 0,1 \\ + \end{smallmatrix} \right]} \alpha A_\alpha$$

In other words, a neutrosophic set can be expressed in terms of the concept of α -cuts without resorting to grade functions μ , $\bar{\sigma}$ and γ . This is what wakes up the representation theorem, and we will leave it at that α -cuts are very convenient for the calculation of the operations and relations equations of neutrosophic sets.

Next let us discuss what is called the extension principle; we will use the functions from X to Y .

Definition 3.5

Extending the function $f : X \rightarrow Y$, the neutrosophic subset A of X is made to correspond to neutrosophic subset $f(A) = (\mu_{f(A)}, \sigma_{f(A)}, \gamma_{f(A)})$ of Y may be the following ways (type1, 2)

- $$\mu_{f(A)}(y) = \begin{cases} \vee \{ \mu_A(x) : x \in f^{-1}(y) \}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$
- $$\sigma_{f(A)}(y) = \begin{cases} \wedge \{ \sigma_A(x) : x \in f^{-1}(y) \}, & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$
- $$\gamma_{f(A)}(y) = \begin{cases} \wedge \{ \gamma_A(x) : x \in f^{-1}(y) \}, & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$
- $$\mu_{f(A)}(y) = \begin{cases} \vee \{ \mu_A(x) : x \in f^{-1}(y) \}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$
- $$\sigma_{f(A)}(y) = \begin{cases} \vee \{ \sigma_A(x) : x \in f^{-1}(y) \}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$
- $$\gamma_{f(A)}(y) = \begin{cases} \wedge \{ \gamma_A(x) : x \in f^{-1}(y) \}, & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$

Let B neutrosophic set in Y . Then the preimage of B , under f , denoted by $f^{-1}(B) = (\mu_{f^{-1}(B)}, \sigma_{f^{-1}(B)}, \gamma_{f^{-1}(B)})$ defined by $\mu_{f^{-1}(B)} = \mu(f(B)), \sigma_{f^{-1}(B)} = \sigma(f(B)), \gamma_{f^{-1}(B)} = \gamma(f(B))$.

Theorem.3.2

Let A, A_i in X , B and $B_j, i \in I, j \in J$ in Y are neutrosophic subsets and $f : X \rightarrow Y$ be a function. Then

- $A_1 \subset A_2 \Rightarrow f(A_1) \subset f(A_2)$.
- $B_1 \subset B_2 \Rightarrow f^{-1}(B_1) \subset f^{-1}(B_2)$,

- $A \subset f(f^{-1}(A))$, the equality holds if f is injective,
- $f(f^{-1}(B)) \subset B$, the equality holds if f is surjective,
- $f^{-1}(\cup_j B_j) = \cup_j f^{-1}(B_j)$,
- $f^{-1}(\cap_j B_j) = \cap_j f^{-1}(B_j)$,
- $f(\cup_i A_i) = \cup_i f(A_i)$,

Proof

Clear.

4- NEUTROSOPHIC IDEALS**Definition.4.1**

Let X is non-empty set and L a non-empty family of NSs. We will call L is a neutrosophic ideal (NL for short) on X if

- $A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],
- $A \in L$ and $B \in L \Rightarrow A \vee B \in L$ [Finite additivity].

A neutrosophic ideal L is called a σ -neutrosophic ideal if $\{A_j\}_{j \in N} \leq L$, implies $\bigvee_{j \in J} A_j \in L$ (countable additivity).

The smallest and largest neutrosophic ideals on a non-empty set X are $\{0_N\}$ and NSs on X . Also, $N.L_f$, $N.L_c$ are denoting the neutrosophic ideals (NL for short) of neutrosophic subsets having finite and countable support of X respectively. Moreover, if A is a nonempty NS in X , then $\{B \in NS : B \subseteq A\}$ is an NL on X . This is called the principal NL of all NSs of denoted by $NL \langle A \rangle$.

Remark 4.1

- If $1_N \notin L$, then L is called neutrosophic proper ideal.
- If $1_N \in L$, then L is called neutrosophic improper ideal.
- $0_N \in L$.

Example.4.1

Any Intuitionistic fuzzy ideal ℓ on X in the sense of Salama is obviously and NL in the form $L = \{A : A = \langle x, \mu_A, \sigma_A, \nu_A \rangle \in \ell\}$

Example.4.2

Let $X = \{a, b, c\}$ $A = \langle x, 0.2, 0.5, 0.6 \rangle$, $B = \langle x, 0.5, 0.7, 0.8 \rangle$, and $D = \langle x, 0.5, 0.6, 0.8 \rangle$, then the family $L = \{0_N, A, B, D\}$ of NSs is an NL on X .

Example.3.3

Let $X = \{a, b, c, d, e\}$ and $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$ given by:

X	$\mu_A(x)$	$\sigma_A(x)$	$\nu_A(x)$
a	0.6	0.4	0.3
b	0.5	0.3	0.3
c	0.4	0.6	0.4
d	0.3	0.8	0.5
e	0.3	0.7	0.6

Then the family $L = \{O_N, A\}$ is an NL on X.

Definition.4.3

Let L_1 and L_2 be two NL on X. Then L_2 is said to be finer than L_1 or L_1 is coarser than L_2 if $L_1 \leq L_2$. If also $L_1 \neq L_2$. Then L_2 is said to be strictly finer than L_1 or L_1 is strictly coarser than L_2 .

Two NL said to be comparable, if one is finer than the other. The set of all NL on X is ordered by the relation L_1 is coarser than L_2 this relation is induced the inclusion in NSs.

The next Proposition is considered as one of the useful result in this sequel, whose proof is clear.

Proposition.4.1

Let $\{L_j : j \in J\}$ be any non - empty family of neutrosophic ideals on a set X. Then $\bigcap_{j \in J} L_j$ and $\bigcup_{j \in J} L_j$ are neutrosophic ideal on X,

In fact L is the smallest upper bound of the set of the L_j in the ordered set of all neutrosophic ideals on X.

Remark.4.2

The neutrosophic ideal by the single neutrosophic set O_N is the smallest element of the ordered set of all neutrosophic ideals on X.

Proposition.4.3

A neutrosophic set A in neutrosophic ideal L on X is a base of L iff every member of L contained in A.

Proof

(Necessity) Suppose A is a base of L. Then clearly every member of L contained in A.

(Sufficiency) Suppose the necessary condition holds. Then the set of neutrosophic subset in X contained in A coincides with L by the Definition 4.3.

Proposition.4.4

For a neutrosophic ideal L_1 with base A, is finer than a fuzzy ideal L_2 with base B iff every member of B contained in A.

Proof

Immediate consequence of Definitions

Corollary.4.1

Two neutrosophic ideals bases A, B , on X are equivalent iff every member of A , contained in B and via versa.

Theorem.4.1

Let $\eta = \left\{ \langle \mu_j, \sigma_j, \gamma_j \rangle : j \in J \right\}$ be a non empty collection of neutrosophic subsets of X . Then there exists a neutrosophic ideal $L(\eta) = \{A \in \text{NSs} : A \subseteq \bigvee A_j\}$ on X for some finite collection $\{A_j : j = 1, 2, \dots, n \subseteq \eta\}$.

Proof

Clear.

Remark.4.3

The neutrosophic ideal $L(\eta)$ defined above is said to be generated by η and η is called sub base of $L(\eta)$.

Corollary.4.2

Let L_1 be an neutrosophic ideal on X and $A \in \text{NSs}$, then there is a neutrosophic ideal L_2 which is finer than L_1 and such that $A \in L_2$ iff $A \vee B \in L_1$ for each $B \in L_2$.

Corollary.4.3

Let $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle \in L_1$ and $B = \langle x, \mu_B, \sigma_B, \nu_B \rangle \in L_2$, where L_1 and L_2 are neutrosophic ideals on the set X . then the neutrosophic set $A * B = \langle \mu_{A*B}(x), \sigma_{A*B}(x), \nu_{A*B}(x) \rangle \in L_1 \vee L_2$ on X where $\mu_{A*B}(x) = \bigvee \{ \mu_A(x) \wedge \mu_B(x) : x \in X \}$, $\sigma_{A*B}(x)$ may be $= \bigvee \{ \sigma_A(x) \wedge \sigma_B(x) \}$ or $\wedge \{ \sigma_A(x) \vee \sigma_B(x) \}$ and $\nu_{A*B}(x) = \wedge \{ \nu_A(x) \vee \nu_B(x) : x \in X \}$.

Theorem.4.2

If L is a neutrosophic ideal on X , then so is L^c is a neutrosophic ideal on X . Where L defined in [7].

Proof

Clear

Theorem.4.3

An NS $L = \left\{ \mathcal{O}_N, \langle \mu_A, \sigma_A, \nu_A \rangle \right\}$ is a neutrosophic ideal on X iff the fuzzy sets μ_A, σ_A and ν_A^c are intuitionistic fuzzy ideals on X .

Proof

Let $L = \left\{ \mathcal{O}_N, \langle \mu_A, \sigma_A, \nu_A \rangle \right\}$ be a NL of X , $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$, then clearly μ_A is a intuitionistic fuzzy ideal on X . Then $\nu_A^c(x) = 1 - \nu_A(x) = \max \left\{ \left(\frac{c}{A} \nu(x), 0 \right) \right\} = \min \left\{ 1, \nu_A^c(x) \right\}$ if $\nu_A^c(x) = \mathcal{O}_N$ then is the smallest intuitionistic fuzzy ideal, or $\nu_A^c(x) = 1_N$ then is the largest intuitionistic fuzzy ideal on X .

Corollary.4.3

L is a neutrosophic ideal on X iff L and $\diamond L$ are neutrosophic ideals on X .

Proof

Clear from the definition 1.3.

Example.4.4

Let L a non empty set and NL on X given by: $L = \{O_N, \langle 0.3, 0.6, 0.2 \rangle, \langle 0.3, 0.5, 0.6 \rangle, \langle 0.2, 0.5, 0.5 \rangle\}$. Then $L = \{O_N, \langle 0.3, 0.7, 0.7 \rangle, \langle 0.2, 0.8, 0.8 \rangle\}$ and $\diamond L = \{O_N, \langle 0.4, 0.6, 0.6 \rangle, \langle 0.5, 0.5, 0.5 \rangle\}$ and $L \subseteq \diamond L$. Where L and $\diamond L$ defined in [7].

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